Overview

For thousands of years, man has used the abacus as a counting and calculating device. No one knows with certainty when or where the abacus was invented, but best scholarship indicates that it was used in Mesopotamia 5 or 6 thousand years ago and was introduced into the Orient through trade with ancient Rome. In the Western world, the abacus gave way to written forms of arithmetic in the 16th or 17th century. It thrived in the Orient and today is common to most Japanese businesses, households, and schools. According to Buddhist tradition, odd numbers are imperfect numbers and even numbers are perfect. The *soroban,* as the abacus is called in Japan, is thought to be an imperfect instrument that is used in the search for perfection.

People with visual impairment or blindness cannot use the abacus that sighted people use, because it has free-moving beads and therefore cannot be read by touch. In the early 1960s, T. V. Cranmer, then director of the Division of Services for the Blind, Kentucky Department of Education, adapted an abacus that blind individuals could use. He added a foam backing to put tension on

the beads and keep them stable. He also increased the length of the rods to give more distance between beads and make them easily read by touch. In the text, *Using the Cranmer Abacus for the Blind,* Fred L. Gissoni (at that time affiliated with the Kentucky Rehabilitation Department) was the first to explain the operation of the Cranmer abacus in 1962. This was followed in 1966 by *The Abacus Made Easy* by Mae E. Davidow, who was at that time teacher of mathematics at the Overbrook School for the Blind. In 1964 at the University of Kentucky, Mr. Gissoni directed the first abacus institute ever held in America. This generated much enthusiasm for the abacus among blind people and teachers of those with visual impairment or blindness; since then, many similar institutes have been held. About the same time, the Hadley School offered a correspondence course in the use of the abacus for blind people throughout the world.

The Cranmer abacus is an efficient and accurate tool that enables you to perform mathematical calculations. It affords more speed and ease of manipulation than braille writers, Taylor slates, pegboards, and other cumbersome tools. It also removes the drudgery formerly associated with arithmetic. Although

calculations can be done quicker on a talking calculator or computer, the abacus is unique as it allows you to mentally perform calculations in an expedient form.

The goal of this course is to provide you with the information you need to become a proficient abacus user. This course is divided into four units. Unit 1, which includes Lessons 1–4, explains how to set, read, and add whole numbers and decimals. Unit 2, which includes Lessons 5–8, examines how to multiply whole numbers and decimals. Unit 3 includes Lessons 9 and 10; it focuses on subtraction of whole numbers and decimals. Lessons 11–15 in the final unit cover long and short division of whole numbers and decimals. This course provides the prerequisite skills that will enable you to proceed to “Abacus 2,” the sequel that features advanced forms of arithmetic. Enrollment in “Abacus 2” is only granted to those who have received their teacher’s recommendation.

To complete this course, you need the materials that The Hadley School for the Blind has provided, including a Cranmer abacus. Additional abacuses can be purchased from the American Printing House for the Blind (APH), 1839 Frankfort Avenue, Louisville,

Kentucky 40206. APH also offers an abacus coupler, which enables you to join two or more abacuses together. In addition, you need writing materials in the medium of your choice. If you are taking the audiocassette version of this course, you also need your own cassette player. Upon completion, you can keep the course materials.

Most lessons in this course feature practice exercises that prepare you to complete the assignment at the end of each lesson. These exercises are for your personal development only, so do **not** mail your answers to your Hadley instructor. Rather, check your comprehension by comparing your answers with those provided.

To help you and your instructor evaluate how well you have mastered the objectives of this course, you must submit the assignment at the end of each lesson. If you are blind or visually impaired, you can mail your assignments as Free Matter for the Blind provided they are in braille or large print (14 point or larger), or on cassette. Sighted students must affix adequate postage. Use the mailing labels enclosed for your convenience. Place the word *Abacus* and your instructor's name in the lower left corner of the

envelope. The enclosed contact information card indicates your instructor's fax number and email address, in case you need to contact your instructor or if you prefer to send your assignments electronically.

Because each lesson builds on previously learned skills, submit only one lesson at a time, at least initially. If you prefer to work at a quicker pace, review this policy with your instructor after completing the first few lessons. Wait for your instructor's feedback before sending in your next assignment so that you can learn from your mistakes.

Now, if you're ready to become a proficient abacus operator, begin Unit 1, Lesson 1: Setting, Reading, and Adding Numbers.

Lesson 1: Setting, Reading, and Adding Numbers

To use the abacus more efficiently and accurately, Lesson 1 defines the special terms associated with this tool. This lesson explains how to set and read numbers, and it introduces addition. Familiarizing yourself with the information in this lesson will enable you to set, read, and add numbers using the abacus.

Objectives

After completing this lesson, you will be able to

a. set and clear numbers
b. read numbers
c. add numbers

Setting Numbers

As with many other specialized tools, the abacus has a special language of its own. To enter a number on the abacus, you do not use the word *write;* instead you *set* a number. To erase or remove a number, use the word *clear.* For instance, to add 7 on a particular column, you'll be told, "Set 7." To subtract 7, you'll be told, "Clear 7." *Adding* is the mathematical process,

whereas *setting* is the operation performed on each column of the abacus to accomplish the addition. For example, consider the addition of 5 plus 3, which equals 8. In order to add 3 to 5, you set three beads on the abacus. Adding refers to the mathematical process, whereas setting means moving the beads.

Occasionally it will be necessary to add or to subtract the number 1 from the column immediately to the left of that upon which you are working. You will refer to this operation as “Set 1 left” or “Clear 1 left.” “Set 1 left” simply means to add the digit 1 on the column immediately left of that upon which you are working. "Clear 1 left" simply means to subtract the digit 1 from the column immediately left of that upon which you are working. If you are told, “Clear 4 and set 1 left,” remove 4 from the column upon which you are working, and set the digit 1 on the column immediately to the left. If you are asked, “Clear 1 left and set 6,” remove 1 from the column immediately to the left of that upon which you are working, and enter 6 on the column upon which you are working.

The beads are used to represent numbers. They take on value when moved toward the bar and lose value when

moved away from it. On each column it is possible to show the digits from zero to nine. However, only one digit can be shown on a given column at one time. Each bead below the separation bar has a value of one. The single bead above the bar has a value of five. When all of the beads on a column are moved away from the bar, that column either contains the digit 0, or it is not being used in the problem and therefore has no value. The single bead above the separation bar, which stands for the digit 5, takes on value when moved down toward the separation bar. It loses its value when moved up away from the separation bar. The four beads below the separation bar each stand for the digit 1. They take on value when moved up toward the separation bar. They lose their value when moved down away from the separation bar.

Generally, numbers should be set and cleared with the right hand. All of the beads below the bar are set with the right thumb and are cleared with the right index finger. The beads above the bar are set and cleared with the right index finger. The index finger of the left hand should always trail the right hand. This will help you keep your place on the proper column and will help to set 1 left or to clear 1 left. As you learn to operate

the abacus, take special care to use the correct finger motions. This will help you attain the greatest possible speed.

Now see how to set and clear the digits from 0 to 9: To set and clear 0, do not change the pattern of beads. All the beads above and below the separation bar should remain away from the separation bar.

To set 1, slide one of the lower beads on the far right column up toward the bar with the right thumb. Clear 1 by sliding this lower bead down with the right index finger.

To set the number 2, slide two lower beads on the far right column up to the bar with the right thumb in a single motion. Clear the number 2 with the right index finger by sliding those two lower beads on the far right column down away from the bar.

To set 3, push three lower beads up to the bar with the right thumb. Clear 3 with the index finger by moving those three lower beads down.

To set 4, slide all four lower beads up to the bar with the thumb. Clear 4 by sliding those four lower beads down with the index finger.

Set 5 with the right index finger by sliding the single upper bead on the far right column down toward the bar. Clear 5 with the index finger by moving that upper bead up away from the bar.

To set 6, use the index finger to move the upper bead down toward the separation bar. Move one lower bead on the same column up with the thumb; 5 plus 1 is 6. To clear 6, move the five bead up with the index finger. Then slide this same finger down across the bar, and move the one lower bead down away from the bar.

To set 7, set the five bead with the index finger, and then set two lower beads with the thumb; 5 plus 2 is 7. Clear 7 by clearing the five bead above the bar with the right index finger. Then move down across the bar and clear the two beads below the bar with the same index finger.

To set 8, first set the five bead with the index finger and set 3 with the thumb. Clear 8 by sliding the five bead up and the three lower beads down, all with the index finger.

To set 9, move all of the beads on the column as close to the bar as possible by sliding the five bead down

with the index finger and all four lower beads up with the thumb. Clear 9 by clearing the five bead and clearing the four lower beads. With practice, the digits 6, 7, 8, and 9 can be set with a single pinching motion. That is, while the index finger is setting the upper five bead, the thumb will be setting the lower beads.

When working with whole numbers, the column to the extreme right end of the abacus is the units column. The next column to its left is the tens column. Then comes the hundreds column, followed by a unit mark or comma that separates the hundreds from the thousands column. Proceeding leftward, you have the thousands, the ten thousands, and the hundred thousands columns. Another comma follows. Then comes the millions column, and so forth, up to the trillions column, which is found on the thirteenth column to the extreme left end of the abacus.

Unit marks are the short, vertical lines found on the separation bar and also at the bottom of the abacus frame. When counting from right to left, these unit marks occur after every third column. Therefore, unit marks are found between columns 3 and 4, columns 6 and 7, columns 9 and 10, and columns 12 and 13. Unit

marks are used as commas or, when necessary, as decimal points.

One column is needed to set a one-digit number, two columns to set a two-digit number, five columns for a five-digit number, ten columns for a ten-digit number, and so on. For example, the number 284 consists of three digits: 2, 8, and 4. So the three far-right columns are needed to set this number. To do so, set the number 2 on the hundreds column (i.e., the third column from the right end), which is known as column 3. Set 8 on the tens column (i.e., the second column from the right end), also known as column 2. Set 4 on the units column (i.e., the far right column of the abacus), which is also called column 1. **Numbers are always set from left to right.** Therefore, anticipate the number of columns needed to set or add a number before you begin.

Try setting the number 13,507. It contains five digits; therefore, it requires five columns to set it. Locate column 5, and set 1 on this column. Set 3 on column 4, followed by a unit mark. Set 5 on column 3, 0 on column 2, and 7 on column 1. When setting or

adding 0, no change in the arrangement of the beads is made on that particular column.

Reading Numbers

Numbers are read in a left-to-right direction. Generally, numbers are read using the thumb, index finger, and middle finger of your right hand. However, develop the method of reading numbers that is most comfortable for you.

Practice setting and reading many numbers. For instance, try setting and reading your phone number, street address, zip code, age, birth year, and historical dates. The current month, day, and year expressed in numerical form can also be set and read on the abacus. Try, for example, setting and reading these numbers.

1. 7
2. 1,067
3. 294
4. 300
5. 9,405
6. 83
7. 246,813
8. 40
9. 4

10. 7,923
11. 81,045
12. 579,108
13. 226
14. 3,336
15. 200,963
16. 10
17. 7,248,019
18. 34,826,120
19. 509
20. 85,003

It is important to set and read numbers accurately and quickly; otherwise the abacus will be of little value to you. Once you can set and read numbers accurately and quickly, you are ready to proceed to addition.

Adding Numbers

Numbers can be added on the abacus by either direct or indirect means. When adding by direct means, simply set the number you want to add on the appropriate column by moving beads representing that number toward the bar. For instance, suppose you want to add 1 plus 2 plus 6. Begin by setting 1 on column 1 by moving one lower bead up to the bar with

the thumb of the right hand. Next add the number 2 by moving two lower beads on the same column up toward the bar. You now have a total of 3 showing on column 1. Now add 6 by sliding the five bead down with the right index finger and by sliding one lower bead up with the thumb. Column 1 now contains the number 9 because 1 plus 2 plus 6 is 9.

Though you should always try to initially add a number directly, it is not always possible to do so. Whenever it is impossible to add a number directly, do so indirectly. Do this by setting either a 5 or a 10. This will result in your adding more than you wanted to add. So subtract the difference between the number you wanted to add and either the 5 or the 10 that you actually added.

To illustrate indirect addition, begin by adding the number 1 to itself a number of times. One is a units number containing a single digit. So all of the 1s must be entered on the units column, which is column 1. Begin by setting the number 1 on column 1 by sliding one of the lower beads up to the bar with the right thumb. (As previously mentioned, your left hand should be resting immediately to the left of the right hand.) Then, on this same column, add 1 to 1 directly. Now

add another 1 directly, and yet another 1. Column 1 now contains the number 4. When you try to add another 1, however, it cannot be added directly. That is, you cannot slide one more lower bead up to the bar because there are no more lower beads to move up. All of them are being used to represent the 4.

However, the five bead on column 1 is not in use. So set 5, which is more than you wanted to add. Recall that you only wanted to add 1, but you have added 5. Since you have over-added, subtract the difference between the 1 (which you wanted to add) and the 5 (which you actually added). That is, subtract 4 because 5 minus 1 equals 4. To do so, move the right index finger above the bar to the top of column 1. Then in one continuous downward sweep of the right index finger, slide the five bead down to the bar, move under it, and slide the four lower beads to the bottom of the column. By doing this, you have indirectly added 1 to 4. Since 4 plus 1 is the same as 4 plus (5 minus 4), you now have the number 5 showing on column 1. This shows one of the indirect methods for adding 1—that is, to set 5 and clear 4.

In this last step, you had to add 1 to 4 indirectly by setting 5 and clearing 4. At first this may seem difficult to understand. But think of it in terms of an everyday money transaction. Instead of exchanging coins, you are exchanging beads. Suppose you have four pennies and a friend owes you one cent. Unfortunately he does not have a penny with him, but he does have a nickel, which he gives to you. Since he gave you 5 cents but owed you only 1 cent, you must give him 4 cents in change. By giving you 5 cents and taking back 4 cents, your friend has indirectly added 1 cent to the 4 cents that you already had. To state this money transaction in the language of the abacus, your friend has indirectly paid you 1 cent by setting 5 cents and clearing 4 cents. You now have a nickel, worth 5 cents.

Now continue to add 1s as you did before. Add 1 to 5 by sliding one lower bead up to the bar with the right thumb. You now have a 5 set above the bar and a 1 immediately below the bar; 5 plus 1 is 6. Now add 1 and you have 7. Add 1 more and you have 8. Add another and you have 9. But when you try to add one more, you cannot add it directly. There are no more inactive (i.e., unused) beads on column 1 to move toward the bar. So with your left hand, set 1 left. Do

this by sliding one lower bead on the column immediately to the left (column 2) up to the bar with the index finger of your left hand.

This one bead equals 10 (i.e., ten 1s) because it is on column 2—the tens column. However, you have over-added—you wanted to add only 1, but you added 10. Therefore, subtract the difference between 10 and 1, which is 9. With the right index finger, clear 9 from column 1. By clearing 9 and setting 1 left, you have indirectly added 1 to 9. The number 10 now appears on columns 2 and 1. The digit 1 appears on column 2 followed by 0 on column 1. This illustrates another rule for the indirect addition of 1, which is to clear 9 and set 1 left.

Think of the last step, 9 plus 1 equals 10, as a money transaction. Now 9 plus 1 is the same as 9 plus (10 minus 9). Suppose you have 9 cents and a friend owes you 1 cent. She does not have a penny, but she does have a dime, which she gives to you. Since she gave you a dime but owed you only 1 cent, she has paid you 9 cents more than she owed you. You must give her 9 cents in change. Your friend has indirectly added 1

cent to your 9 cents by setting 10 cents and clearing 9 cents. You now have a dime, worth 10 cents.

Now continue to add 1s on column 1. Add 1 to 10 by sliding one lower bead up to the bar on column 1. This produces one bead immediately below the bar on both columns 2 and 1, which represent the number 11. Now continue to add 1 directly until you reach 14. To do so, add 1 to 14 indirectly. Add 1 to the 4 of 14, indirectly, by setting 5 and clearing 4 with a continuous downward sweep of the right index finger. Your total then will be 15.

When you reach 19, use indirect addition to add 1 to the 9 of 19. Because the five bead is already in use, you cannot set 5 and clear 4. So clear 9 and set 1 left. Clear 9 on column 1 with the right hand, and set 1 left on column 2 with the left hand. At this point, the abacus will have a 2 on column 2, and a 0 on column 1, which together represent the number 20.

Now continue to add 1 on column 1 until you reach 49. To add 1 to 49, add 1 to the 9 of 49 on column 1. You cannot add 1 directly; nor can you set 5 and clear 4, because the five bead is in use. So clear 9 and set 1 left. Clear 9 from column 1 and set 1 left on column 2.

However, column 2 already contains a 4. Because there are no more lower beads on column 2 to move up to the bar, it is not possible to set 1 left directly. So set 1 left indirectly by setting 5 and clearing 4 with a continuous downward sweep of the left index finger. In this operation, you have actually cleared 9, set 50, and cleared 40, thereby indirectly adding 1 to 49. Your abacus should now contain 50.

Continue to add 1 on column 1 until you reach 99. To add 1 to 99, add 1 to the 9 of 99 on column 1. This cannot be done directly; nor can you set 5 and clear 4, because the five bead is in use. So add 1 to 9 indirectly by clearing 9 and setting 1 left. Clear 9 from column 1 with the right hand and set 1 left on column 2. Since column 2 contains a 9, it is impossible to set 1 left directly. Therefore, with the left hand, clear 9 from column 2, move leftward to column 3, and set 1 on that column. By doing this, you have actually cleared 9, cleared 90, and set 100, thereby adding 1 to 99 indirectly. Your abacus now contains 100 on columns 3, 2, and 1.

Summary

This lesson introduced the terminology you need for using the abacus. It explained how the beads represent numbers, and how to set and read those numbers. The lesson also described how to add numbers.

Assignment 1

Complete this assignment in the medium of your choice. Begin by giving your full name, address, and phone number. Also indicate the course title, Assignment 1, your instructor's name, and the date. Then provide your answers. Be sure to indicate the question number along with each answer. Note that each question is worth 6 2/3 points. Instructions for sending assignments can be found in the Overview to the course.

Indicate whether the following statements are true or false. If the statement is false, reword it to make it true.

1. Blind or visually impaired people can easily and effectively use the same abacus that sighted people use.

2. Numbers are set and read from right to left.

3. The separation bar appears in the lower third of the abacus.

4. The unit marks are the short vertical lines appearing after every fifth column.

5. To set a number means to enter it on the abacus; to clear a number means to remove it.

6. Beads take on their value when they are moved as close to the separation bar as possible.

7. If a column already contains the digit 4, the digit 1 can be added to it directly.

8. An unused column and one that contains the digit 0 look and feel the same.

9. The individual bead on each column, either above or below the separation bar, has a value of 5.

Select the answer that best completes the following:

10. To add 1 to 64, you must
    a. add 1 to 6 directly
    b. add 1 to 4 indirectly by setting 5 and clearing 4
    c. add 1 to 4 indirectly by clearing 9 and setting 1 left

11. How many beads are needed to show the digit 7?
    a. three
    b. one
    c. seven

12. How many columns are needed to set 7,205?
    a. three
    b. four
    c. five

13. You cannot add 1 directly to
    a. 0
    b. 5
    c. 9

14. Set 12 on the two columns to the far right. To add 1, you must
    a. add it directly to the 1 of 12
    b. add it directly to the 2 of 12
    c. add it indirectly to the 2 of 12

15. Set 49 on the two columns to the far right. To add 1, you must
    a. add it directly to the 9 of 49
    b. add 1 to the 4 in column 2
    c. add 1 indirectly to the 9 in column 1 by first clearing 9, and then setting 1 left indirectly, by setting 5 and clearing 4

Once you have completed your assignment, mail, fax, or email it to your instructor. Although you may

proceed to Lesson 2, wait for your instructor's feedback before submitting your next assignment.